Information filtering by smart nodes in random networks

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Diffusion of information in social networks has drawn extensive attention from various scientific communities, with many contagion models proposed to explain related phenomena. In this paper, we present a simple contagion mechanism, in which a node will change its state immediately if it is exposed to the diffusive information. By considering two types of nodes (smart and normal) and two kinds of information (true and false), we study analytically and numerically how smart nodes influence the spreading of information, which leads to information filtering. We find that for randomly distributed smart nodes, the spreading dynamics over random networks with Poisson degree distribution and power-law degree distribution (with relatively small cutoffs) can both be described by the same approximate mean-field equation. Increasing the heterogeneity of the network may elicit more deviations, but not much. Moreover, we demonstrate that more smart nodes make the filtering effect on a random network better. Finally, we study the efficacy of different strategies of selecting smart nodes for information filtering.

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I. INTRODUCTION

Prevalent viewpoints, ideas, or rumors are all examples of information spreading, which has attracted a great deal of attention from researchers in various areas, including social science and statistical physics [1–12]. It is commonly believed that human interactions play the key role in the information diffusion process. Due to communications, information transfers from one person to another, which may cause a large scale of awareness or infection [13].

Many models have been proposed to explain information diffusion phenomena [14], such as epidemic models [15–21], rumor-spreading models [22–25], cascading models [26,27], and threshold models [28–35]. In these models, individuals are classified by their different states, where actually the interaction patterns are quite different. For example, in the SI epidemic model [or the Daley-Kendal (DK) rumor-spreading model] [14,22], each interaction between susceptible (ignorant) and infected (spreader) nodes is independent, and it sustains until one of the two nodes switches its state. In the cascading models, however, an active node has only one chance to affect its inactive neighbors (the interactions are also independent). The contagion mechanism underlying these models is “simple,” implying that a node may change its state even if there is only one active neighbor. However, it fails to characterize some social spreading processes, such as the spreading of behaviors or innovations, where social reinforcement should be taken into account [4,8]. The linear threshold model [3], first proposed by Granovetter, is the standard tool to capture these “complex” contagion phenomena. It assumes that a person will change its state only if a certain fraction of its neighbors have done so. This model has been studied extensively after the stimulating work of Watts [28].

The aforementioned models have been studied extensively in the scenario of one piece of information. However, people are inundated with miscellaneous information in everyday life. Some information is true, which is valuable to us. For example, the information of individual states during an epidemic outbreak over a social network can spur more people to take precautionary measures, which will suppress the transmission of the disease [36–39]. However, some information is false, which may cause problems for people, such as online rumors. Yet, to filter out false information is a rather challenging task. In one sense, personal abilities play a crucial role in the filtering process. For instance, some individuals have critical-thinking abilities (being “smart”), and they usually perform better than the normal ones in discriminating false information. How do different kinds (true or false) of information spread in the presence of smart people? This is an interesting question that needs to be fully addressed in order to better understand various complex phenomena observed from the real information diffusion processes in human society.

In this paper, we present a different but simple (thus analytically tractable) contagion network model in which nodes have three different states: 0 (susceptible), 1 (adopted), or 2 (immune). We assume that a susceptible node will change its state (to the adopted or immune state) immediately after one of its neighbors has become adopted. Motivated by observations in real life, we incorporate smart agents into the model, after which we study the differences in spreading true and false information on random networks. Our aim is to understand how the number of smart nodes and their distribution affect the filtering of information, where true information can spread...
throughout the system while false information will be purged. Though the model is simple, it may provide enough insights for more complex and realistic situations.

We organize the paper as follows. In Sec. II, we present the details of our model. In Sec. III, we consider the case of a random distribution of smart nodes in the network, and we study analytically and numerically how these smart nodes influence the filtering of information. In Sec. IV, we compare three different distribution strategies of selecting smart nodes for information filtering. Finally, we make a conclusion in Sec. V.

II. MODEL

The model consists of $N$ nodes forming a random network with degree distribution $p(k)$. The average degree is $z = \Sigma_k kp(k)$. A node in the network can be in one of the three states: 0 (susceptible), 1 (adopted), or 2 (immune). Initially all nodes are in state 0, except that one node is selected as the seed at random, which is in state 1. At each time step, all nodes are updated synchronously [40]. If a node with state 0 has no adopted neighbors, it remains in state 0 at the next step. Yet, if there is at least one adopted node around it, the node will change its state according to the following rule:

$$s_i(t + 1) = \begin{cases} 
1 & \text{with probability } p, \\
2 & \text{with probability } 1 - p, 
\end{cases}$$

where $s_i(t + 1)$ represents the state of node $i$ at time $t + 1$. Rule (1) means that once a susceptible node is exposed to any adopted node, it will become adopted with probability $p$ or refuse with probability $1 - p$. Once a node becomes adopted or immune, it cannot change its state until the end of the process. It is worth noting that we only consider two possible state transitions in the spreading dynamics ($0 \rightarrow 1$ and $0 \rightarrow 2$). Different from the traditional rumor-spreading model (and the SIR epidemiological model), the transition of state from 1 to 2 is neglected. This dynamics is suitable to characterize behaviors such as reposting an article or a message on a social website—once a person reposts it, he/she will probably keep it and thus always be in an adopted state. As a supplement, the transition $1 \rightarrow 2$ is also considered in a modified model as shown in the Appendix.

In real life, both true and false information exists. Facing this, individuals exhibit heterogeneous abilities of discrimination of false information. For example, some people are well educated or have critical-thinking skills, so they could distinguish whether a message is true or false with ease. To mimic such a situation, we select a fraction $r$ of the nodes in the network as smart nodes (agents), who have different adopting probabilities from the normal ones. We assume that the probability of a normal node to become adopted is $p_0$ (regardless of true or false information). The probability of a smart node becoming adopted is

$$p_s = \begin{cases} 
H_1 & \text{true message}, \\
H_0 & \text{false message}, 
\end{cases}$$

where $H_1 > p_s > H_0$. This indicates that, for the true (false) information, a smart node will adopt it with a higher (lower) probability than a normal node.

III. RANDOM DISTRIBUTION OF SMART NODES

For the sake of mathematical simplicity, in this section we first assume that the smart nodes are randomly distributed on the network, that is, a randomly chosen node has a probability $r$ being smart, which will be a controlling parameter in our model. We provide a mean-field analysis for the spreading dynamics in Sec. III A, and we confirm the theoretical predictions by extensive simulations on ER networks and an uncorrelated configuration model (UCM) with degree distribution $p(k) \sim k^{-2} e^{-k/\zeta}$ in Sec. III B.

A. Theoretical analysis

We take the mean-field approximation approach to investigating the spreading dynamics analytically. Supposing that $\rho_{\infty}$ is the adoption density at the steady state, we have the following equation:

$$\rho_{\infty} = \rho_0 + (1 - \rho_0) \sum_{k=1}^{\infty} p(k) \left[ \sum_{m=1}^{k} r p_s b_{k,m}(\rho_{\infty}) \right] 
+ \sum_{m=1}^{k} (1 - r) p_n b_{k,m}(\rho_{\infty}) \equiv F(\rho_{\infty}),$$

where $b_{k,m}(\rho_{\infty}) = \binom{k}{m} \rho_{\infty}^m (1 - \rho_{\infty})^{k-m}$, denoting the probability of a node of degree $k$ has $m$ adopted neighbors at $t = \infty$, and $\rho_0 = 1/N$ is the initial adoption density, which goes to 0 if the network size is infinitely large. The probability that a randomly chosen node is in state 1 at time $t = \infty$ is the sum of two contributions: the probability $\rho_0$ that the chosen node is adopted at $t = 0$ and the probability $1 - \rho_0$ that the node is susceptible at $t = 0$ but has at least one adopted neighbor at time $t = \infty$. Since the chosen node is either smart or normal, the probability that it adopts is the sum of two parts: $\sum_{m=1}^{k} r p_s b_{k,m}(\rho_{\infty})$ and $\sum_{m=1}^{k} (1 - r) p_n b_{k,m}(\rho_{\infty})$. The sum over $k$ accounts for all possible degrees a node may have.

The value of $\rho_{\infty}$ can be obtained by solving Eq. (3) iteratively. Moreover, the global cascade condition (as $N \rightarrow \infty$, $\rho_{\infty}$ corresponds to a finite value) can also be determined from this equation. Note that $\rho_{\infty} = 0$ is always a solution to $\rho_{\infty} = F(\rho_{\infty})$ (in the case of $\rho_0 \rightarrow 0$). To have a positive solution, the condition $F(\rho_{\infty})|_{\rho_0=0} > 1$ must be fulfilled. Thus we get

$$z[r p_s + (1 - r) p_n] > 1.$$  \hfill (4)

Note that, when $z < 1$, inequality (4) can never be satisfied [as $r p_s + (1 - r) p_n < 1$]. In this case, both true and false messages cannot spread out, since the network is under percolation. We are interested in the case of $z > 1$, assuming that the network is well connected.

Our aim is to find a parameter regime in which the true message can spread widely through the system while the false message cannot. For the true message ($p_s = H_1$), the condition for global cascading is

$$(H_1 - p_n) r_T + p_n > 1/z.$$  \hfill (5)
are the simulation results for $N = 5000$ and the average degree is $z$. We choose $H_1 = 2p_n$ and $H_0 = 0$ as assumed in the theoretical analysis. All the results are averaged over 1000 different realizations.

Figure 1 shows the global cascading boundary in the $(p_n, r)$ space for both true and false information. The blue circles and yellow squares correspond to $z = 5$ and 10, respectively. Taking $z = 5$ for example, there are two boundary lines (the left and right ones represent the spreading of true and false information, respectively), which separate the phase diagram into three regions. In region I, both true and false information cannot spread out. Even if the network is full of smart nodes, the contagion probability ($p_n$ or $p_s$) is too small to trigger a large number of nodes to adopt. In region II, the true information can percolate through the whole network, while the false information cannot. This is an ideal region in which the network can filter out the true message from mixed ones. In region III, both true and false information could spread throughout the network. It can be seen that, when $p_n$ is greater than a specific value ($p_n > 1$ for $z = 5$), there appears a threshold $r_c$, for each $p_n$ that separates two different phases, indicating that it requires at least $r_c$ smart nodes to successfully filter different messages. As $p_n$ increases, $r_c$ drops at first and then grows again. The decreasing behavior is natural since larger $p_n$ (or $p_s$) makes the true information easier to spread even among normal nodes. However, when $p_n$ increases further, the false information could also trigger global cascading. To suppress this effect, more smart nodes are needed to hinder the spreading process.

The increase of the average degree promotes the spreading of both true and false information. As shown in Fig. 1 (comparing the red solid lines and the green dashed lines), the boundary lines are shifted to the left. However, the influence of the average degree on the spread of false information is more profound than that of true information (for $r > 0$). Note that the smart nodes make the false messages hard to spread; in this case, increasing the average number of links of a node could effectively break this constraint. On the contrary, for true messages, even without many links, the smart nodes could help the information spread out successfully, which makes the average degree less important. It is obvious that, as the average degree increases, the filtering region shrinks.

Figure 2 illustrates the asymptotic density $\rho_\infty$ of adopted nodes as a function of $p_n$ for different values of $r$ with $z = 10$. The solid and dashed lines are the theoretical solutions, respectively. The points are obtained when the final density $\rho_\infty$ of adopted nodes exceeds 0.5% of the population.

For the false message ($p_s = H_0$), the condition for a note to be in the absorbing state (no global cascades) is

\[(H_0 - p_n)r_F + p_n < 1/z. \tag{6}\]

Without loss of generality, we assume that $H_1 = 2p_n$ and $H_0 = 0$, satisfying $H_1 > p_n > H_0$. Thus, the above inequalities become

\[r_T > \Phi = \frac{1/z - p_n}{p_n}, \tag{7}\]

\[r_F > -\Phi = \frac{1/z - p_n}{-p_n}. \tag{8}\]

Denote $\rho^T$ and $\rho^F$ as the final adoption densities for true and false information, respectively. The above inequalities imply three typical regions for the spreading dynamics: (i) $0 < p_n < z/\Phi$ ($\Phi > 1$). In this case, $\rho^T \to 0$ and $\rho^F \to 0$. (ii) $z/\Phi \leq p_n \leq 1/z$ ($0 \leq \Phi \leq 1$). In this case, inequality (8) is always satisfied, meaning that $\rho^F \to 0$, while for the true information, there is a threshold $r_c = \Phi$, below which $\rho^T \to 0$, but for $r > r_c$, $\rho^T \to 0$. (iii) $p_n > 1/z$ ($-1 < \Phi < 0$). The system is always in the active state for true messages, i.e., $\rho^T > 0$. But for the false information there is a threshold $r_c = -\Phi$, which separates the two different situations.

**B. Simulation results**

We first perform simulations on uniformly distributed random networks [41]. The size of the underlying network is $N = 5000$ and the average degree is $z$. The results are averaged over 1000 different realizations.

Figure 1 shows the global cascading boundary in the $(p_n, r)$ space for both true and false information. The blue circles and yellow squares correspond to $z = 5$ and 10, respectively. Taking $z = 5$ for example, there are two boundary lines (the left and right ones represent the spreading of true and false information, respectively).
Moreover, the asymptotic values of the density of adopted nodes grows, the density of adopted nodes at the final state for various $\rho_\infty$ exceeds 0.5% of the population. We then turn to heterogeneous random networks. Consider the degree distribution $p(k) \sim k^{-\lambda} e^{-k/\kappa}$, where $\lambda$ and $\kappa$ are constants. These kinds of networks (having a power-law degree distribution with an exponential cutoff) are ubiquitous in nature, including collaboration networks, e-mail networks, and protein networks [42,43]. Unlike the uniform random networks, where the degrees of most nodes are around the average value, the degree distributions of heterogeneous random networks are highly skewed. In this case, the majority of nodes have small degrees while a few have large numbers of connections. In the simulations we take the UCM networks, where the degrees of most nodes are around the average value, the degree distributions of heterogeneous random networks are highly skewed. In this case, the majority of nodes have small degrees while a few have large numbers of connections. In the simulations we take the UCM networks, with the maximal degree satisfying $k_{\text{max}} < N^{1/2}$ and the minimal degree being $k_{\text{min}} = 3$ [44]. We show the phase diagrams on heterogeneous networks in Fig. 3. The red circles correspond to the UCM networks with $\kappa = 12$ (i.e., $z \approx 5$). The solid lines correspond to Eqs. (7) and (8) for $z = 5$. We can see that the deviations are small, indicating that the mean-field method also works for heterogeneous networks (with relatively small $\kappa$) approximately. Moreover, the asymptotic values of the density of adopted nodes $\rho_\infty$ for various $p_n$ almost overlap in UCM and ER networks with $z = 5$ [see Fig. 4(a)]. Note that for ER networks, the variance of degree $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = z$. The variance of degree for UCM networks with $\kappa = 12$ is 12.58, which is more than twice that of ER networks with $z = 5$. Increasing the heterogeneity of the node degree may make the deviations become larger, but not too much. As shown in Fig. 3, the green squares and dashed lines correspond to the UCM networks with $\kappa = 100$ ($z \approx 6.9, \sigma^2 = 55.43$) and the mean-field solutions for $z = 6.9, \sigma^2 = 55.43$ and the mean-field solutions for $z = 6.9$. Correspondingly, the values of $\rho_\infty$ for various $p_n$ in two kinds of networks are shown in Fig. 4(b), which deviate only a little. Further increasing $\kappa$ has similar conclusions [45]. These arguments suggest that our model is quite different from the traditional epidemic models (and the threshold models), where the spreading process is highly sensitive to the hubs. In our model, the role of hubs in the spreading of information seems to be weakened. To better understand why this is, we take the star networks as an extreme example. As a comparison, the case of ER networks with the same average degree is also considered [Fig. 5(a)]. For simplicity, we assume all nodes are normal here. Initially, a node is selected randomly as a seed, and the adoption probability is set to be $p_n = 0.2$. The size distribution of adopted nodes in star networks is bimodal [as shown in Fig. 5(b)], indicating that there is a small probability that a randomly chosen seed may trigger a large cascade ($s \approx 1000$). However, in ER networks, the largest cascade size is much smaller ($s \approx 16$). This means that the hubs could greatly facilitate the spreading of information. On the other hand, the probability that a randomly chosen seed triggers a very small cascade (with size $s = 1$) in star networks is higher than that in ER networks. Once the hubs turn into state 2 (with probability $1 - p_n$), a large number of nodes cannot be affected by the information. From this point of view, the hubs may also impede the spreading of information. As a consequence, the two actions might offset (at least partly) each other.

**IV. SELECTING STRATEGIES FOR SMART NODES**

The selection of smart nodes in the network plays an important role in the filtering process. To elucidate the effect of different distributions of smart nodes in a network, we compare...
three heuristic strategies of selection: (i) random selection, which we already studied in Sec. III; (ii) selecting nodes according to their degrees, i.e., sorting the nodes by degree and selecting the \( r N \) largest nodes as smart ones; (iii) selecting nodes by \( k \)-shell index in descending order. The selection of smart nodes in (ii) and (iii) may not be unique—there may be many sets of nodes that can be selected, since some nodes have the same degree or \( k \)-shell index. In such situations, we choose one at random.

Figure 6 shows the phase diagrams for the three selecting strategies on ER networks of size \( N = 5000 \) with average degree \( z = 5 \). It can be seen that the highest-degree selection strategy performs best, which corresponds to the largest filtering region (region II). Specifically, for a fixed value of \( p_n \), this selection method requires the fewest smart nodes for the outbreak of the true information and extinction of the false information. Since the nodes with a high degree can influence a large number of nodes in the network, choosing these nodes as smart ones has a better performance than the case of random selection. The \( k \)-shell method performs worse than the highest-degree selection strategy but better than the random one. Note that, statistically, higher \( k \)-shell nodes may also have higher degrees. Nevertheless, it cannot guarantee that their degrees are always the highest. On the other hand, nodes in the highest \( k \)-shells, by construction, are connected to other high-degree nodes [34]. This implies that smart nodes tend to affect smart nodes (if one selects smart nodes by the descending order of their \( k \)-shell index), which confines the efficacy of smart nodes in the filtering process.

V. CONCLUSION

In this paper, we have proposed a simple contagion model incorporating smart nodes to study how different kinds (true or false) of information spread in networked populations. In our model, a node would take action (adopt or refuse) immediately as soon as it is exposed to the diffusive information. We argue that this contagion mechanism is suited to modeling such behaviors as reposting messages to social websites, where people are more concerned about the content of the messages and ignore what others do (peer pressure). Based on this observation, we considered how true and false information spreads in the presence of smart nodes, who have a higher (lower) probability to adopt true (false) information. We have shown that, in the \( (p_n, r) \) parameter space, where \( p_n \) is the normal adoption probability and \( r \) is the fraction of smart nodes, there is a filtering region in which true information can percolate throughout the network while false information cannot. For ER networks with a random distribution of smart nodes, the spreading dynamics can be described by a mean-field equation. We found that this equation also works approximately for heterogeneous networks with relatively small cutoffs. Increasing the heterogeneity of the network may elicit deviations (relatively small). These results indicate that, in our model, the spreading of information is not very sensitive to the hubs. The reason lies in the double-edged role the hubs may play: if they adopt, many nodes could be influenced by the diffusive information, which facilitates the spreading process; if they refuse to adopt, many nodes would be ignorant of the information, which hinders the spreading of information. Finally, we studied the effect of different selection strategies for smart nodes on the information spreading process. Our results demonstrate that selecting smart nodes by their degrees (from the highest down) works best, which corresponds to the largest filtering region in the phase diagram. This model may help us to better understand some complex spreading phenomena in real life. Moreover, it also has the potential to measure the information filtering ability of a social network, and, furthermore, to improve the filtering performance. This model can also be modified by considering more intriguing...
cases, for instance the recurrent activation process [46–48], which will need many further investigations.

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APPENDIX: A MODIFIED MODEL

In the previous model, there are only two state transitions: 0 → 1 and 0 → 2. However, in some scenarios (especially for traditional rumor-spreading), it is also possible for the transition 1 → 2 to happen, since when people know the news they may lose interest in spreading it (because the rumor has lost its “new value”). We assume that, at each time step, a susceptible node will become adopted with probability 𝜌 or immune with probability 1 − 𝜌 if there is (at least) one adopted neighbor; in the meantime, the adopted nodes will become immune with probability 𝛼 if they encounter another node in an adopted or immune state. Figure 7 shows the phase diagrams for the modified model for different values of 𝛼. Similar to the previous model, we find that the phase diagram is divided into three regions. Moreover, it seems that the parameter 𝛼 has no effect on the cascading dynamics, since the time for a node to turn into the immune state is inessential in the model.

FIG. 7. Phase diagrams for the modified model. The circles and squares correspond to 𝛼 = 0.1 and 0.2, respectively. We perform simulations on ER networks with an average degree 𝜇 = 5. These results are obtained when 𝜌∞ (the density of immune nodes) exceeds 0.5% of the population.
[40] We also performed simulations using the asynchronous updating rule and got the same results.
[45] We calculated the case of $\kappa = 1000$ for UCM networks (the average degree $z \approx 7.47$ and the variance of degree is 74.36) and got similar conclusions.