Sinusoidal Steady-State Analysis

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Structure

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Why Sinusoidal Source?

• The generation, transmission, distribution, and consumption of electric energy occur under essentially sinusoidal steady-state conditions.

• An understanding of sinusoidal behavior makes it possible to predict the behavior of circuits with nonsinusoidal sources.

• Steady-state sinusoidal behavior often simplifies the design of electrical systems.

A Household Distribution Circuit
A sinusoidal voltage/current source (independent or dependent) produces a voltage/current that varies sinusoidally with time.

\[ v = V_m \cos (\omega t + \phi) \]

- \( V_m \): Maximum amplitude
- \( \omega \): Angular frequency
- \( \phi \): Phase angle (radians/degrees)

\[ \omega = 2\pi f = \frac{2\pi}{T} \]

- \( f \): Frequency
- \( T \): Period

\( \frac{180^\circ}{\pi} \) (number of degrees) = (number of radians)
RMS Value

The **rms** value of a periodic function is defined as the square root of the mean value of the squared function.

\[
V_{\text{rms}} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) \, dt}
\]

We can completely describe a specific sinusoidal signal if we know its **frequency**, **phase angle**, and **amplitude** (either the maximum or the rms value).
Example #1

A sinusoidal current has a maximum amplitude of 20 A. The current passes through one complete cycle in 1 ms. The magnitude of the current at zero time is 10 A.

a) What is the frequency of the current in hertz?

b) What is the frequency in radians per second?

c) Write the expression for $i(t)$ using the cosine function. Express $\phi$ in degrees.

d) What is the rms value of the current?
Solution for Example #1

a) From the statement of the problem, $T = 1$ ms; hence $f = 1/T = 1000$ Hz.

b) $\omega = 2\pi f = 2000\pi$ rad/s.

c) We have $i(t) = I_m \cos(\omega t + \phi) = 20 \cos(2000\pi t + \phi)$, but $i(0) = 10$ A. Therefore $10 = 20\cos \phi$ and $\phi = 60^\circ$. Thus the expression for $i(t)$ becomes

$$i(t) = 20\cos(2000\pi t + 60^\circ).$$

d) The rms value of a sinusoidal current is $I_m/\sqrt{2}$. Therefore the rms value is $20/\sqrt{2}$, or 14.14 A.
Example #2

- Calculate the rms value of the periodic triangular current in the given figure. Express your answer in terms of the peak current $I_p$. 

![Diagram of a periodic triangular current wave]
Solution for Example #2

\[ I_{\text{rms}} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} i^2 \, dt} \]

\[
\int_{t_0}^{t_0+T} i^2 \, dt = 4 \int_{0}^{T/4} i^2 \, dt \\
i = \frac{4I_p}{T} t, \quad 0 < t < T/4
\]

\[
\int_{t_0}^{t_0+T} i^2 \, dt = 4 \int_{0}^{T/4} \frac{16I_p^2}{T^2} t^2 \, dt = \frac{I_p^2 T}{3}
\]

\[ I_{\text{rms}} = \frac{I_p}{\sqrt{3}} \]
The Sinusoidal Response

\[ v_s = V_m \cos (\omega t + \phi) \]

\[ L \frac{di}{dt} + Ri = V_m \cos (\omega t + \phi) \]

\[ i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos (\phi - \theta) e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos (\omega t + \phi - \theta) \]

Transient component

Steady-state component
Steady-state Solution

• The steady-state solution is a sinusoidal function.
• The frequency of the response signal is identical to the frequency of the source signal. *This condition is always true in a linear circuit when the circuit parameters, \( R, L, \) and \( C \), are constant.*
• The maximum amplitude of the steady-state response, in general, differs from the maximum amplitude of the source. For the circuit being discussed, the maximum amplitude of the response signal is \( V_m/\sqrt{(R^2+\omega^2L^2)} \), and that of the signal source is \( V_m \).
• The phase angle of the response signal, in general, differs from the phase angle of the source. For the circuit being discussed, the phase angle of the current is \( \phi - \theta \) and that of the voltage source is \( \phi \).
The Phasor

The **phasor** is a complex number that carries the **amplitude** and **phase angle** information of a sinusoidal function (please see the Appendix B).

**Euler’s identity:**

\[
e^{\pm j\theta} = \cos \theta \pm j \sin \theta \quad \text{The real part} \\
\cos \theta = \Re \{e^{j\theta}\} \\
\sin \theta = \Im \{e^{j\theta}\} \quad \text{The imaginary part}
\]

\[
v = V_m \cos (\omega t + \phi) = V_m \Re \{e^{j(\omega t + \phi)}\} = \Re \{V_m e^{j\phi} e^{j\omega t}\}
\]

**Phasor transform:**

\[
V = V_m e^{j\phi} = \mathcal{P}\{V_m \cos (\omega t + \phi)\} \quad \text{Angle notation} \quad 1/e^{j\phi} = 1/\phi^\circ
\]

\[
V = V_m \cos \phi + j V_m \sin \phi
\]
Inverse Phasor Transform

\[ \mathbf{v} = 100 \angle -26^\circ \quad \Rightarrow \quad v = 100\cos(\omega t - 60) \]

Inverse phasor transform:

\[ \mathcal{P}^{-1}\{V_m e^{j\phi}\} = \Re\{V_m e^{j\phi} e^{j\omega t}\} \]

The phasor transform is useful in circuit analysis because it reduces the task of finding the maximum amplitude and phase angle of the steady-state sinusoidal response to the algebra of complex numbers.
• The transient component vanishes as time elapses, so the **steady-state component of the solution must also satisfy the differential equation**.

• In a linear circuit driven by sinusoidal sources, the steady-state response also is sinusoidal, and **the frequency of the sinusoidal response is the same as the frequency of the sinusoidal source**.

• Using the notation introduced in Eq. 9.11, we can postulate that the steady-state solution is of the form \( R\{Ae^{j\beta}e^{j\omega t}\} \), where \( A \) is the maximum amplitude of the response and \( \beta \) is the phase angle of the response.

• When we substitute the postulated steady-state solution into the differential equation, the exponential term \( e^{j\omega t} \) cancel out, leaving the solution for \( A \) and \( \beta \) in the domain of complex numbers
Illustration

\[ i_{ss}(t) = \Re \{ I_m e^{j\beta} e^{j\omega t} \} \]

\[ \Re \{ j\omega LI_m e^{j\beta} e^{j\omega t} \} + \Re \{ RI_m e^{j\beta} e^{j\omega t} \} = \Re \{ V_m e^{j\phi} e^{j\omega t} \} \]

\[ \Re \{ (j\omega L + R)I_m e^{j\beta} e^{j\omega t} \} = \Re \{ V_m e^{j\phi} e^{j\omega t} \} \]

\[ (j\omega L + R)I_m e^{j\beta} = V_m e^{j\phi} \]

\[ I_m e^{j\beta} = \frac{V_m e^{j\phi}}{R + j\omega L} \]
The phasor transform, along with the inverse phasor transform, allows you to go back and forth between the time domain and the frequency domain. Therefore, when you obtain a solution, you are either in the time domain or the frequency domain. You cannot be in both domains simultaneously. Any solution that contains a mixture of time domain and phasor domain nomenclature is nonsensical.

The phasor transform is also useful in circuit analysis because it applies directly to the sum of sinusoidal function, if

\[ v = v_1 + v_2 + \cdots + v_n \]

When all the voltages on the right-hand side are sinusoidal voltages of the same frequency, then

\[ V = V_1 + V_2 + \cdots + V_n \]
Example #3

If \( y_1 = 20\cos(\omega t - 30^\circ) \) and \( y_2 = 40\cos(\omega t + 60^\circ) \), express \( y = y_1 + y_2 \) as a single sinusoidal function.

a) Solve by using trigonometric identities.

b) Solve by using the phasor concept.
Solution for Example #2

\[ y = y_1 + y_2 \quad \Rightarrow \quad Y = Y_1 + Y_2 \]

\[ = 20/\angle -30^\circ + 40/\angle 60^\circ \]

\[ = (17.32 - j10) + (20 + j34.64) \]

\[ = 37.32 + j24.64 \]

\[ = 44.72/\angle 33.43^\circ \]

\[ y = \mathcal{P}^{-1}\{44.72e^{j33.43}\} = \Re\{44.72e^{j33.43}e^{j\omega t}\} \]

\[ = 44.72 \cos (\omega t + 33.43^\circ) \]
The Passive Circuit Elements in the Frequency Domain

The V-I Relationship for a Resistor

\[ v = R[I_m \cos(\omega t + \theta_i)] = RI_m[\cos(\omega t + \theta_i)] \]

\[ V = RI_m e^{j\theta_i} = RI_m/\theta_i \]

\[ V = RI \]

At the terminals of a resistor, there is no phase shift between the current and voltage.
The V-I Relationship for an Inductor

\[ v = L \frac{di}{dt} = -\omega LI_m \sin(\omega t + \theta_i) \]

\[ v = -\omega LI_m \cos(\omega t + \theta_i - 90^\circ) \]

\[ V = -\omega LI_m e^{j(\theta_i - 90^\circ)} = -\omega LI_m e^{j\theta_i} e^{-j90^\circ} = j\omega LI_m e^{j\theta_i} = j\omega LI \]

\[ V = (\omega L \angle 90^\circ) I_m \angle \theta_i = \omega LI_m \angle (\theta_i + 90^\circ) \]
The V-I Relationship for a Capacitor

\[ i = C \frac{dv}{dt} \]

\[ v = V_m \cos(\omega t + \theta_v) \]

\[ I = j \omega C V \]

\[ V = \frac{1}{j \omega C} I \]

\[ V = \frac{1}{\omega C} \sqrt{-90^\circ} I_m \angle \theta_i = \frac{I_m}{\omega C} \angle (\theta_i - 90^\circ) \]
Impedance and Reactance

\[ V = Z I \]

Impedance: measured in ohms
Reactance: the imaginary part of impedance

<table>
<thead>
<tr>
<th>Circuit Element</th>
<th>Impedance</th>
<th>Reactance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>( R )</td>
<td>(-)</td>
</tr>
<tr>
<td>Inductor</td>
<td>( j\omega L )</td>
<td>( \omega L )</td>
</tr>
<tr>
<td>Capacitor</td>
<td>( j(-1/\omega C) )</td>
<td>(-1/\omega C)</td>
</tr>
</tbody>
</table>
Kirchhoff’s law in the Frequency Domain

\[ v_1 + v_2 + \cdots + v_n = 0 \]

\[ V_{m_1} \cos(\omega t + \theta_1) + V_{m_2} \cos(\omega t + \theta_2) + \cdots + V_{m_n} \cos(\omega t + \theta_n) = 0 \]

\[ \Re \{ V_{m_1} e^{j\theta_1} e^{j\omega t} \} + \Re \{ V_{m_2} e^{j\theta_2} e^{j\omega t} \} + \cdots + \Re \{ V_{m_n} e^{j\theta_n} e^{j\omega t} \} \]

\[ \Re \{ (V_{m_1} e^{j\theta_1} + V_{m_2} e^{j\theta_2} + \cdots + V_{m_n} e^{j\theta_n}) e^{j\omega t} \} = 0 \]

\[ \Re \{ (\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n) e^{j\omega t} \} = 0 \]

\[ \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0 \quad \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0 \]
Series, Parallel, and Delta-to-Wye Simplification

The rules for combining impedances in series or parallel and for making delta-to-wye transformations are the same as those for resistors. The only difference is that combining impedances involves the algebraic manipulation of complex numbers.

**Impedances in series:**

\[ V_{ab} = Z_1 I + Z_2 I + \cdots + Z_n I \]
\[ = (Z_1 + Z_2 + \cdots + Z_n)I \]

\[ Z_{ab} = \frac{V_{ab}}{I} = Z_1 + Z_2 + \cdots + Z_n \]

**Electric Circuits**
**Impedance in Parallel**

\[ I = I_1 + I_2 + \cdots + I_n \]

\[ \frac{V}{Z_{ab}} = \frac{V}{Z_1} + \frac{V}{Z_2} + \cdots + \frac{V}{Z_n} \]

\[ \frac{1}{Z_{ab}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n} \]

\[ Z_{ab} = \frac{Z_1 Z_2}{Z_1 + Z_2} \]

**Susceptance**

\[ Y = \frac{1}{Z} = G + jB \text{ (siemens)} \]

\[ Y_{ab} = Y_1 + Y_2 + \cdots + Y_n \]

**Electric Circuits**
Delta-to-Wye Transformations

\[ Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} \]

\[ Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2} \]

\[ Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} \]

Electric Circuits

\[ Z_1 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c} \]

\[ Z_2 = \frac{Z_cZ_a}{Z_a + Z_b + Z_c} \]

\[ Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c} \]
Example #4

A 90 Ω resistor, a 32 mH inductor, and a 5 μF capacitor are connected in series across the terminals of a sinusoidal voltage source. The steady-state expression for the source voltage $v_s$ is $750 \cos (5000t + 30^\circ)$ V.

a) Construct the frequency-domain equivalent circuit.

b) Calculate the steady-state current $i$ by the phasor method.
Solution for Example #4

\[ \omega = 5000 \text{ rad/s} \]

\[ Z_L = j\omega L = j(5000)(32 \times 10^{-3}) = j160 \Omega \]

\[ Z_C = j \frac{1}{\omega C} = -j \frac{10^6}{(5000)(5)} = -j40 \Omega \]

\[ V_s = 750/30^\circ \text{ V} \]

\[ Z_{ab} = 90 + j160 - j40 = 90 + j120 = 150/53.13^\circ \Omega \]

\[ I = \frac{750/30^\circ}{150/53.13^\circ} = 5/23.13^\circ \text{ A} \]

\[ i = 5 \cos (5000t - 23.13^\circ) \text{ A} \]

Electric Circuits
Source Transformations and Thévenin-Norton Equivalent Circuits

![Diagram of Thévenin and Norton equivalents]

- **Frequency-domain linear circuit**: may contain both independent and dependent sources.

**The Node-Voltage Method (Example)**

**The Mesh-Current Method**
Example #5

Use the node-voltage method to find the branch currents $I_a$, $I_b$, and $I_c$ in the circuit.

Electric Circuits
Solution for Example #5

**Node 1:** 
\[-10.6 + \frac{V_1}{10} + \frac{V_1 - V_2}{1 + j2} = 0\]

**Node 2:** 
\[\frac{V_2 - V_1}{1 + j2} + \frac{V_2}{-j5} + \frac{V_2 - 20I_x}{5} = 0\]

\[
I_x = \frac{V_1 - V_2}{1 + j2}
\]

\[
V_1 = 68.40 - j16.80 \text{ V}
\]

\[
V_2 = 68 - j26 \text{ V}
\]
\[ I_a = \frac{V_1}{10} = 6.84 - j1.68 \, \text{A}, \]
\[ I_x = \frac{V_1 - V_2}{1 + j2} = 3.76 + j1.68 \, \text{A}, \]
\[ I_b = \frac{V_2 - 20I_x}{5} = -1.44 - j11.92 \, \text{A}, \]
\[ I_c = \frac{V_2}{-j5} = 5.2 + j13.6 \, \text{A}. \]

Check

\[ I_x = I_b + I_c \]
The Transformer

A transformer is a device that is based on magnetic coupling. Transformers are used in both communication and power circuits.

- **linear transformer**: is found primarily in communication circuits
- **Ideal transformer**: is used to model the ferromagnetic transformer found in power systems.
A simple **transformer** is formed *when two coils are wound on a single core to ensure magnetic coupling*.

- **Primary winding**: the transformer winding connected to the source;
- **Secondary winding**: the winding connected to the load as the.

\[
R_1 = \text{the resistance of the primary winding} \\
R_2 = \text{the resistance of the secondary winding} \\
L_1 = \text{the self-inductance of the primary winding} \\
L_2 = \text{the self-inductance of the secondary winding} \\
M = \text{the mutual inductance}
\]
\[ V_s = (Z_s + R_1 + j\omega L_1)I_1 - j\omega M I_2 \]

\[ 0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2 \]

\[ Z_{11} = Z_s + R_1 + j\omega L_1 \]

\[ Z_{22} = R_2 + j\omega L_2 + Z_L \]

\[ I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_s \]

\[ I_2 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2} V_s = \frac{j\omega M}{Z_{22}} I_1 \]

\[ V_s \frac{1}{I_1} = Z_{\text{int}} = \frac{Z_{11}Z_{22} + \omega^2 M^2}{Z_{22}} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} \]

\[ Z_{ab} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \]

The impedance \( Z_{ab} \) is independent of the magnetic polarity of the transformer!
Reflected Impedance

\[ Z_{ab} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \]

\[ Z_L = R_L + jX_L \]

\[ Z_r = \frac{\omega^2 M^2}{R_2 + R_L + j(\omega L_2 + X_L)} = \frac{\omega^2 M^2[(R_2 + R_L) - j(\omega L_2 + X_L)]}{(R_2 + R_L)^2 + (\omega L_2 + X_L)^2} = \frac{\omega^2 M^2}{|Z_{22}|^2} [(R_2 + R_L) - j(\omega L_2 + X_L)] \]
The Ideal Transformer

An ideal transformer consists of two magnetically coupled coils having $N_1$ and $N_2$ turns, respectively, and exhibiting these three properties:
1. The coefficient of coupling is unity ($k = 1$).
2. The self-inductance of each coil is infinite ($L_1 = L_2 = \infty$).
3. The coil losses, due to parasitic resistance, are negligible.
Exploring Limiting Values

\[ Z_{ab} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \]

\[ Z_{22} = R_2 + R_L + j(\omega L_2 + X_L) = R_{22} + jX_{22} \]

\[ Z_{ab} = R_1 + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} + j\left(\omega L_1 - \frac{\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}\right) = R_{ab} + jX_{ab} \]

\[ M^2 = L_1 L_2 \]

\[ X_{ab} = \omega L_1 - \frac{(\omega L_1)(\omega L_2)X_{22}}{R_{22}^2 + X_{22}^2} = \omega L_1\left(1 - \frac{\omega L_2 X_{22}}{R_{22}^2 + X_{22}^2}\right) \]

\[ X_{22} = \omega L_2 + X_L \]

\[ X_{ab} = \omega L_1\left(\frac{R_{22}^2 + \omega L_2 X_L + X_L^2}{R_{22}^2 + X_{22}^2}\right) \]
Factoring $\omega L_2$ out of the numerator and denominator yields:

$$X_{ab} = \frac{L_1}{L_2} \frac{X_L + (R_{22}^2 + X_L^2)/\omega L_2}{(R_{22}/\omega L_2)^2 + [1 + (X_L/\omega L_2)]^2}$$

$L_1 \rightarrow \infty$, $L_2 \rightarrow \infty$, and $k \rightarrow 1.0$  \hspace{1cm} L_1/L_2 = (N_1/N_2)^2$

$$X_{ab} = \left(\frac{N_1}{N_2}\right)^2 X_L \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} = \frac{L_1}{L_2} R_{22} = \left(\frac{N_1}{N_2}\right)^2 R_{22}$$

$$Z_{ab} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 + \left(\frac{N_1}{N_2}\right)^2 (R_L + jX_L)$$

Scaling factor
Determining the Voltage and Current Ratios

(a) \[ I_1 = \frac{V_1}{j\omega L_1} \]
\[ V_2 = j\omega M I_1 \]
\[ \frac{V_1}{N_1} = \frac{V_2}{N_2} \]
\[ L_1/L_2 = \left(\frac{N_1}{N_2}\right)^2 \]
\[ V_2 = \sqrt{\frac{L_2}{L_1}} V_1 \]
\[ M^2 = L_1 L_2 \]

(b) \[ 0 = -j\omega M I_1 + j\omega L_2 I_2 \]
\[ \frac{I_1}{I_2} = \frac{L_2}{M} = \frac{L_2}{\sqrt{L_1 L_2}} = \sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1} \]
\[ I_1 N_1 = I_2 N_2 \]

Electric Circuits

R_1 = R_2 = 0
Determining the Polarity of the Voltage and Current Ratios

a. If the coil voltages $V_1$ and $V_2$ are both positive or negative at the dot-marked terminal, use a plus sign. Otherwise, use a negative sign.

b. If the coil currents $I_1$ and $I_2$ are both directed into or out of the dot-marked terminal, use a minus sign. Otherwise, use a plus sign.

\[
\begin{align*}
\frac{V_1}{N_1} &= \frac{V_2}{N_2}, \\
N_1I_1 &= -N_2I_2
\end{align*}
\]

(a) \hspace{2cm} \frac{V_1}{N_1} = -\frac{V_2}{N_2}, \\
N_1I_1 &= N_2I_2

(b) \hspace{2cm} \frac{V_1}{N_1} = \frac{V_2}{N_2}, \\
N_1I_1 &= -N_2I_2

(c) \hspace{2cm} \frac{V_1}{N_1} = \frac{V_2}{N_2}, \\
N_1I_1 &= N_2I_2

(d) \hspace{2cm} \frac{V_1}{N_1} = -\frac{V_2}{N_2}, \\
N_1I_1 &= -N_2I_2

Electric Circuits
\[ a = \frac{N_2}{N_1} \]

Important parameter for ideal transformer!
Ideal Transformer Used for Impedance Matching

Thus, the ideal transformer's secondary coil reflects the load impedance back to the primary coil, with the scaling factor $1/a^2$.

Note that the ideal transformer changes the magnitude of $Z_L$ but does not affect its phase angle. Whether $Z_{IN}$ is greater or less than $Z_L$ depends on the turns ratio $a$. 

Electric Circuits
Example #6

The load impedance connected to the secondary winding of the ideal transformer consists of a 237.5 mΩ resistor in series with a 125 µH inductor.

If the sinusoidal voltage source \((v_g)\) is generating the voltage \(2500 \cos 400t\) V, find the steady-state expressions for: (a) \(i_1\); (b) \(v_1\); (c) \(i_2\); and (d) \(v_2\).
Solution for Example #6

\[ 2500 \angle 0^\circ = (0.25 + j2)I_1 + V_1 \]

\[ V_1 = 10V_2 = 10[(0.2375 + j0.05)I_2] \]

\[ I_2 = 10I_1 \]

\[ I_1 = 100 \angle -16.26^\circ \text{ A} \]

\[ V_1 = 2500 \angle 0^\circ - (100 \angle -16.26^\circ)(0.25 + j2) \]

\[ = 2420 - j185 = 2427.06 \angle -4.37^\circ \text{ V} \]

\[ i_1 = 100 \cos (400t - 16.26^\circ) \text{ A} \]

\[ v_1 = 2427.06 \cos (400t - 4.37^\circ) \text{ V} \]
Phasor Diagrams

Constructing phasor diagrams of circuit quantities generally involves both currents and voltages. As a result, two different magnitude scales are necessary, one for currents and one for voltages.

The ability to visualize a phasor quantity on the complex-number plane can be useful when you are checking pocket calculator calculations.
Example #7

• For the given circuit, use a phasor diagram to find the value of $R$ that will cause the current through that resistor, $i_R$, to lag the source current, $i_s$, by 45° when $\omega = 5$ krad/s.

![Phasor Diagram](image)
Solution for Example #7

\[ I_L = \frac{V_m \angle 0^\circ}{j(5000)(0.2 \times 10^{-3})} = V_m \angle -90^\circ \]

\[ I_C = \frac{V_m \angle 0^\circ}{-j/(5000)(800 \times 10^{-6})} = 4V_m \angle 90^\circ \]

\[ I_R = \frac{V_m \angle 0^\circ}{R} = \frac{V_m}{R} \angle 0^\circ \]

\[ R = 1/3 \Omega \]
## Summary

<table>
<thead>
<tr>
<th>Element</th>
<th>Impedance ((Z))</th>
<th>Reactance</th>
<th>Admittance ((Y))</th>
<th>Susceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>(R) (resistance)</td>
<td>()</td>
<td>(G) (conductance)</td>
<td>()</td>
</tr>
<tr>
<td>Capacitor</td>
<td>(j(-1/\omega C))</td>
<td>(-1/\omega C)</td>
<td>(j\omega C)</td>
<td>(\omega C)</td>
</tr>
<tr>
<td>Inductor</td>
<td>(j\omega L)</td>
<td>(\omega L)</td>
<td>(j(-1/\omega L))</td>
<td>(-1/\omega L)</td>
</tr>
</tbody>
</table>